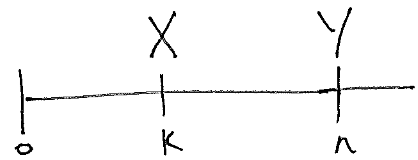
1/14/20

M1S1: Accumulation Function

$a(t) = paf_0^t$



$Y = X \cdot paf_k^n = X \cdot \frac{a(n)}{a(k)}$

$X = Y \cdot pdf_n^k = Y \cdot \frac{a(k)}{a(n)}$

M1S2: Simple & Compound Interest

i - simple interest $\Longrightarrow a(t) = 1 + i \cdot t$
(t - in years)

i - periodic effective interest rate $\Longrightarrow a(t) = (1+i)^t$
(t - # of periods)

Examples:

1) $i = .06$ simple $(\Rightarrow a(t) = 1 + .06t)$

$$pdf_4^1 = \frac{a(1)}{a(4)} = \frac{1.06}{1.24} = .8548\ldots$$

$$pdf_8^5 = \frac{a(5)}{a(8)} = \frac{1.3}{1.48} = .8783\ldots$$

2) $i = .06$ annual effective interest rate (aeir)

$a(t) = 1.06^t$ (t - in years)

$$pdf_4^1 = \frac{a(1)}{a(4)} = \frac{1.06}{1.06^4} = (1.06)^{-3} = .8396\ldots$$

$$pdf_8^5 = \frac{a(5)}{a(8)} = \frac{1.06^5}{1.06^8} = (1.06)^{-3} = .8396\ldots$$

$=$

"v-notation" v = the periodic discount factor
$= \frac{1}{1+i}$ i = periodic eir

In this example $i = .06 = aeir$

$$\Rightarrow v = \frac{1}{1.06} = adf$$

<u>Note:</u> $v^{-1} = 1.06 = aaf$

$$v^2 = \left(\frac{1}{1.06}\right)^2 = badf \quad \text{bi-annual discount factor}$$

$$v^{-1/2} = saf = (1.06)^{1/2}$$

MIS3: Simple & Compound Discount

d - simple discount rate $\Rightarrow a(t) = (1-dt)^{-1}$ (t - in years)

d - periodic effective discount rate

$$\Rightarrow a(t) = (1-d)^{-t}$$

<u>Example:</u> $d = .06$ simple $\Rightarrow a(t) = (1-.06t)^{-1}$

$$pdf_{10}^{4} = \frac{a(4)}{a(10)} = \frac{.76^{-1}}{.4^{-1}} = \frac{.4}{.76} = .5263\ldots$$

$$pdf_{5}^{1} = \frac{a(1)}{a(5)} = \frac{.94^{-1}}{.7^{-1}} = \frac{.7}{.94} = .7446\ldots$$

$d = .06$ aedr $\Rightarrow a(t) = (.94)^{-t}$

$$pdf_{K+1}^{K} = \frac{a(k)}{a(k+1)} = \frac{(.94)^{-K}}{(.94)^{-(K+1)}} = \frac{.94^{K+1}}{.94^{k}} = .94$$

$$v = .94 = adf = 1 - aedr$$

When compounding

i = peir – periodic effective interest rate

d = pedr – ______________ discount rate

paf – periodic accumulation factor

pdf – periodic discount factor

$$v^{-1} = \text{paf} = 1+i = (1-d)^{-1}$$

$$v = \text{pdf} = 1-d = (1+i)^{-1}$$